The First Note book

First prep

The Rational Numbers

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Set of Rational numbers

Remember:-

1) Counting Numbers = $\{1, 2, 3, 4, \dots\}$

2) Set of Integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

3) Set of Natural Number = $\{0, 1, 2, 3, \dots\}$

4) $Z^+ = \{1, 2, 3, \dots\}$

5) $Z^- = \{-1, -2, -3, \dots\}$

6) $N \subset Z$

7) $Z = Z^+ \cup \{0\} \cup Z^-$

8) If $|x| = a$ then $x = \pm a$

Ex 1:- Complete

1) $Z - N = \dots$

2) $Z^+ \cup \{0\} = \dots$

3) $Z^+ \cap Z^- = \dots$

4) $N \cap Z =$

5) $|-3| = \dots$

6) $|0| = \dots$

7) If $|x| = 7$ then $x = \dots$ or $x = \dots$

8) If $|x| = 0$ then $x = \dots$

Ex 2: find the value of x when

1) $x + 2 = |-3|$

2) $|x| + 2 = 5$

3) $|x| - 5 = 2 + |-2|$

4) $2x + 5 = 7$

Definition of the rational number:-

It's a number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$

i.e.

$$Q = \{x : x = \frac{a}{b}, a \in Z, b \in Z, b \neq 0\}$$

Ex:- put $\in$ or $\notin$

		the answer	Because
[1] $\frac{2}{3}$ ...	Q	$\in$	$\frac{2}{3}$ in the form $\frac{a}{b}, b \neq 0$
[2] 0.2 ...	Q	$\in$	$0.2 = \frac{2}{10}$ in the form $\frac{a}{b}, b \neq 0$
[3] 25% ...	Q	$\in$	$25\% = \frac{25}{100}$ in the form $\frac{a}{b}$
[4] $1\frac{1}{4}$ ...	Q	$\in$	$1\frac{1}{4} = \frac{5}{4}$ in the form $\frac{a}{b}$
[5] -2 ...	Q	$\in$	$-2 = \frac{-2}{1}$
[6] Zero ...	Q	$\in$	$\text{zero} = \frac{0}{1}$
[7] 3 ...	Q	$\in$	$3 = \frac{3}{1}$
[8] $\frac{1}{0}$ ...	Q	$\notin$	because the denominator = 0
[9] $\frac{2}{5-5}$ ...	Q	$\notin$	because the denominator = 0

Notes

[1] $\frac{a}{b} \in Q$ if $b \neq 0$ (denominator = zero)

[2] $\frac{a}{b} = 0$ if $a = \text{zero}$, $b \neq \text{zero}$

[3] $\frac{a}{b}$ is an integer if the numerator is divisible by the denominator.

4) $\frac{a}{b} > 0$ (positive) if $ab > 0$

(if a, b have the Same sign)

5) $\frac{a}{b} < 0$ (Negative) if $ab < 0$

Ex:- If x is an integer, write the required condition to make each of the following a rational number:

1) $\frac{2}{x-5}$ ⇒ the Condition $x - 5 \neq 0$ i.e $x \neq 5$

2) $\frac{5}{x+3}$ ⇒ the Condition $x + 3 \neq 0$ i.e $x \neq -3$

3) $\frac{7}{2x-4}$ ⇒ the Condition $2x - 4 \neq 0$ ⇒ $2x \neq 4$ $\div 2$

i.e. $x \neq 2$

4) $\frac{5}{2x}$ ⇒ the Condition $2x \neq 0$ $\div 2$ i.e $x \neq 0$

5) $\frac{3}{|x|-4}$ ⇒ the Condition $|x| - 4 \neq 0$ ⇒ $|x| \neq 4$

i.e. $x \neq \pm 4$

6) $\frac{2}{3x+6}$ ⇒ the Condition $3x + 6 \neq 0$

therefore ⇒ $3x \neq -6$ $\div 3$

i.e. $x \neq -2$

Ex:- If x is an integer, write the required condition to mak each of the following = Zero

1) $\frac{x-3}{4}$

Since $\frac{x-3}{4}=0$ therefore $x-3=0$ i.e $x=3$

2) $\frac{2-x}{3}$

Since $\frac{2-x}{3}=0$ there fore $2-x=0$ i.e. $x=2$

3) $\frac{x+3}{x+4}$

Since $\frac{x+3}{x+4}=0$ therefore $x+3=0$ i.e. $x=-3$

4) $\frac{3x-3}{x}$

Since $\frac{3x-3}{x}=0$ therefore $3x-3=0$

$\Rightarrow 3x=3 \quad \div 3$ i.e $x=1$

5) $\frac{2x}{x+3}$

Since $\frac{2x}{x+3}=0$ therefore $2x=0 \quad \div 2 \Rightarrow x=0$

6) $\frac{3x+6}{x+1}$

Since $\frac{3x+6}{x+1}=0$ therefore

$3x+6=0$

$3x=-6 \quad \div 3$

$x=-2$

Different forms of arational number

(1) Writing the rational number in its simplest form:

Ex: put each of the following numbers in its simplest form:

(1) $\frac{10}{15} = \frac{2}{3}$ $(\div 5)$ Dividing the two terms by 5

(2) $\frac{45}{20} = \frac{9}{4}$ $(\div 5)$

(3) $\left| -\frac{18}{12} \right| = \frac{18}{12} = \frac{3}{2}$ Dividing the two Terms by 6

(4) $|20\%| = \frac{20}{100} = \frac{1}{5}$ Dividing the two terms by 20

(5) $-\frac{24}{28} = -\frac{6}{7}$ Dividing the two Terms by 4

* the form of aterminating decimal

To write the rational number in the form of aterminating decimal we mak its denominator equal to 10, 100, 1000, ...

Ex: write each of the following numbers in the form of aterminating decimal:

(1) $\frac{2}{5}$ (To mak denominator 10 multiplying the two terms by 2

$\frac{2}{5} = \frac{4}{10} = 0.4$

2) $-\frac{3}{8}$

To make denominator 1000 multiplying the two terms by 125)

$-\frac{3}{8} = \frac{-3 \times 125}{8 \times 125} = -\frac{375}{1000} = -0.375$

3) $\frac{3}{4}$

To make denominator 100 multiplying the two Terms by 25

$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$

4) $\frac{7}{25}$

To make denominator 100 multiplying the two terms by 4

$\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$

5) $-2\frac{2}{25} = -2\frac{2 \times 4}{25 \times 4} = -2\frac{8}{100} = -2.08$

6) $\frac{7}{5} = \frac{7 \times 2}{5 \times 2} = \frac{14}{10} = 1.4$

7) $|-2\frac{1}{5}| = 2\frac{1}{5} = 2\frac{1 \times 2}{5 \times 2} = 2\frac{2}{10} = 2.2$

8) $-5\frac{3}{5} = -5\frac{3 \times 2}{5 \times 2} = -5\frac{6}{10} = -5.6$

* The form of percentage :

To write the rational number in the form of percentage we make its denominator = 100

Ex: Write each of the following numbers in the form of percentage

1) $\frac{3}{5} = \frac{3}{5} \times 100\% = 60\%$

2) $\frac{7}{5} = \frac{7}{5} \times 100\% = 140\%$

3) $\frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$

4) $\frac{5}{4} = \frac{5}{4} \times 100\% = 125\%$

5) $\left|-\frac{6}{25}\right| = \frac{6}{25} \times 100\% = 24\%$

6) $3.2 = 3.2 \times 100\% = 320\%$

7) $0.15 = 0.15 \times 100\% = 15\%$

8) $\frac{9}{20} = \frac{9}{20} \times 100\% = 45\%$

The recurring decimal

Ex:- Write each of the following in the form $\frac{a}{b}$

1) $0.\dot{5} = \frac{5}{9}$

2) $-0.\dot{7} = -\frac{7}{9}$

3) $0.\dot{4}\dot{5} = \frac{45}{99} = \frac{5}{11}$

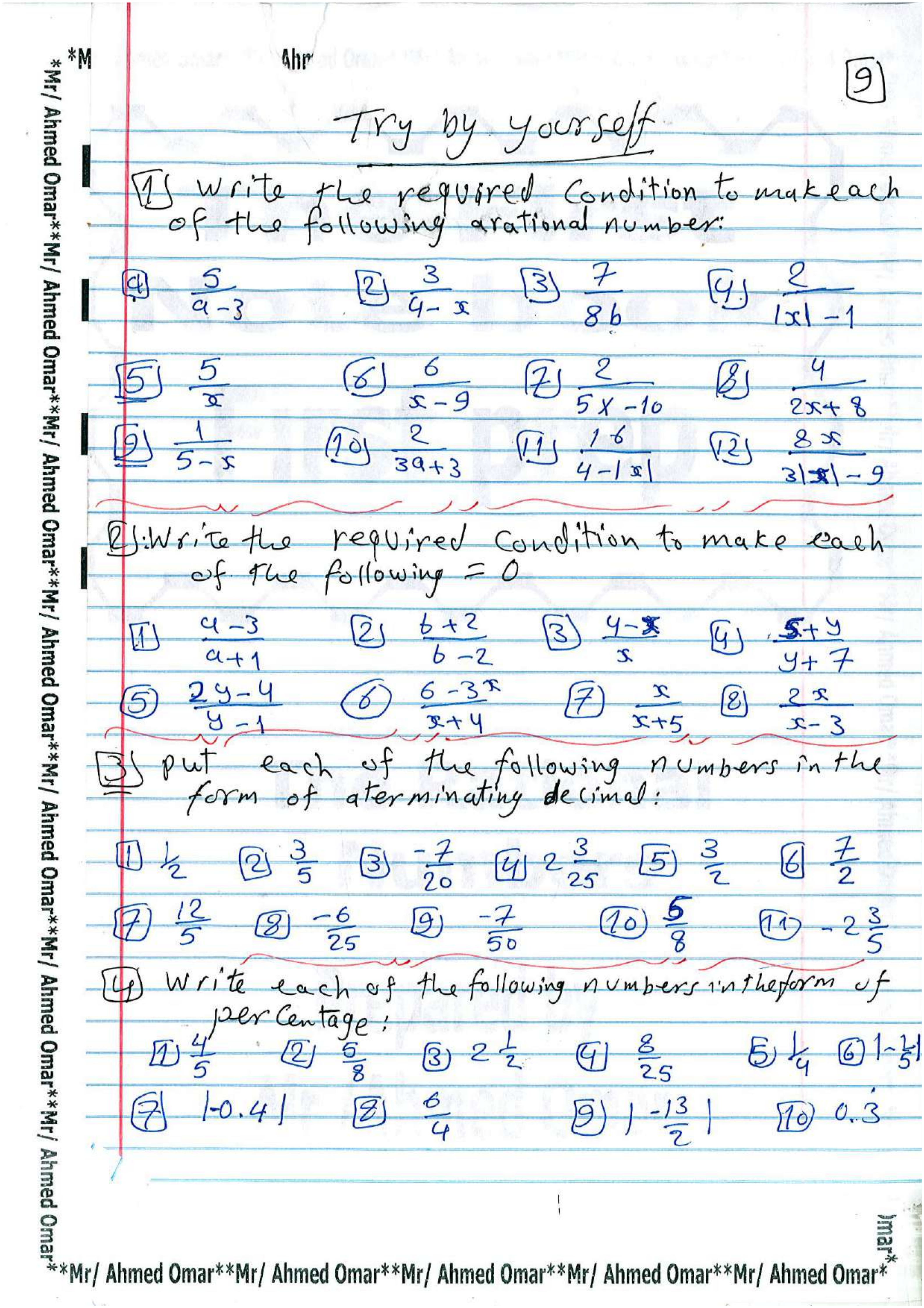
4) $-0.\dot{1}\dot{2} = -\frac{12}{99} = -\frac{4}{33}$

5) $0.\dot{4}0\dot{7} = \frac{407}{999}$

6) $9.\dot{3} = 9\frac{3}{9} = 9\frac{1}{3}$

Ex:- Write each of the following in the form $\frac{a}{b}$

1. $0.4 = \frac{4}{10} = \frac{2}{5}$
2. $0 = \frac{0}{1}$
3. $-0.01 = -\frac{1}{100}$
4. $0.75 = \frac{75}{100} = \frac{3}{4}$
5. $3\frac{3}{4} = \frac{15}{4}$
6. $8\frac{2}{3} = \frac{26}{3}$
7. $0.001 = \frac{1}{1000}$
8. $-7 = \frac{-7}{1}$
9. $1.25 = \frac{125}{100} = \frac{25}{20} = \frac{5}{4}$
10. $30\% = \frac{30}{100} = \frac{3}{10}$
11. $45\% = \frac{45}{100} = \frac{9}{20}$
12. $0.\dot{5} = \frac{5}{9}$
13. $0.\dot{1}\dot{5} = \frac{15}{99} = \frac{5}{33}$
14. $-1.\dot{1}\dot{8} = -1\frac{18}{99} = -1\frac{2}{11} = \frac{-13}{11}$
15. $0.0\dot{4}\dot{5} = \frac{45}{990} = \frac{5}{110} = \frac{1}{22}$
16. $0.\dot{2}8\dot{5} = \frac{285}{999} = \frac{95}{333}$



Try by yourself

1) Write the required condition to make each of the following a rational number:

1) $\frac{5}{a-3}$ 2) $\frac{3}{4-x}$ 3) $\frac{7}{8b}$ 4) $\frac{2}{|x|-1}$

5) $\frac{5}{x}$ 6) $\frac{6}{x-9}$ 7) $\frac{2}{5x-10}$ 8) $\frac{4}{2x+8}$

9) $\frac{1}{5-x}$ 10) $\frac{2}{3a+3}$ 11) $\frac{16}{4-|x|}$ 12) $\frac{8x}{3|x|-9}$

2) Write the required condition to make each of the following $= 0$

1) $\frac{a-3}{a+1}$ 2) $\frac{b+2}{b-2}$ 3) $\frac{y-x}{x}$ 4) $\frac{5+y}{y+7}$

5) $\frac{2y-4}{y-1}$ 6) $\frac{6-3x}{x+4}$ 7) $\frac{x}{x+5}$ 8) $\frac{2x}{x-3}$

3) put each of the following numbers in the form of a terminating decimal:

1) $\frac{1}{2}$ 2) $\frac{3}{5}$ 3) $\frac{-7}{20}$ 4) $2\frac{3}{25}$ 5) $\frac{3}{2}$ 6) $\frac{7}{2}$

7) $\frac{12}{5}$ 8) $\frac{-6}{25}$ 9) $\frac{-7}{50}$ 10) $\frac{5}{8}$ 11) $-2\frac{3}{5}$

4) Write each of the following numbers in the form of percentage:

1) $\frac{4}{5}$ 2) $\frac{5}{8}$ 3) $2\frac{1}{2}$ 4) $\frac{8}{25}$ 5) $\frac{1}{4}$ 6) $|-\frac{1}{5}|$

7) $|-0.4|$ 8) $\frac{6}{4}$ 9) $|-\frac{13}{2}|$ 10) $0.\dot{3}$

Comparing and ordering Rational Numbers.

Ex.1 Represent the rational number $\frac{3}{4}$ on the number line

Solution:-

-1 0 a 1

$\frac{-5}{4}$ $\frac{-4}{4}$ $\frac{-3}{4}$ $\frac{-2}{4}$ $\frac{-1}{4}$ $\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\boxed{\frac{3}{4}}$ $\frac{4}{4}$ $\frac{5}{4}$

Ex.2: Represent the rational numbers $2, 0, \frac{3}{2}, \frac{-5}{2}, -3$ on the number line then arrange them descendingly

Solution:

We should convert their denominators to have common denominator at first

Since L.C.M of the denominators is 2

Then the numbers after converting their denominators are

$\frac{4}{2}, \frac{0}{2}, \frac{3}{2}, \frac{-5}{2}, \frac{-6}{2}$

$\frac{-7}{2}$ $\frac{-6}{2}$ $\frac{-5}{2}$ $\frac{-4}{2}$ $\frac{-3}{2}$ $\frac{-2}{2}$ $\frac{-1}{2}$ $\frac{0}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ $\frac{3}{2}$ $\frac{4}{2}$ $\frac{5}{2}$ $\frac{6}{2}$

then ; $\frac{4}{2} > \frac{3}{2} > \frac{0}{2} > \frac{-5}{2} > \frac{-6}{2}$

i.e $2 > \frac{3}{2} > 0 > \frac{-5}{2} > -3$

Ex3 Compare between the two rational numbers $\frac{2}{3}$ and $\frac{3}{4}$

Solution

We should convert their denominators to have common denominator at first

$\frac{2}{3}$ ① ② ③ $\frac{3}{4} \Rightarrow \frac{8}{12} \quad \frac{9}{12}$

Since $\frac{8}{12} < \frac{9}{12}$ then $\frac{2}{3} < \frac{3}{4}$

Ex4: find four rational numbers lying between the two numbers ① $\frac{1}{2}$ and $\frac{1}{3}$

② $\frac{-3}{5}$ and $\frac{-2}{3}$

Solution

① $\frac{1}{2} \quad \frac{1}{3} \Rightarrow \frac{3}{6}$ and $\frac{2}{6}$

$\Rightarrow \frac{30}{60}$ and $\frac{20}{60}$

$\Rightarrow \frac{20}{60} < \frac{21}{60} < \frac{22}{60} < \frac{23}{60} < \frac{24}{60} < \frac{30}{60}$

② $\frac{-3}{5} \quad \frac{-2}{3} \Rightarrow -\frac{9}{15} \quad -\frac{10}{15}$

$\Rightarrow \frac{-90}{150} \quad -\frac{100}{150}$

then the numbers are: $-\frac{91}{100}$, $-\frac{92}{100}$, $-\frac{93}{100}$, $-\frac{94}{100}$

Ex: 5 Complete each of the following using the suitable Sign (<, > or =):

1) $\frac{7}{5}$ ... $\frac{4}{5}$ 2) $-\frac{3}{4}$ ... $-\frac{2}{4}$

3) $\frac{1}{5}$ ... $\frac{1}{6}$ 4) $\frac{3}{6}$ ... $\frac{2}{3}$

5) $\frac{4}{10}$ ... $\frac{14}{35}$ 6) $|-\frac{10}{15}|$ ... $\frac{2}{3}$

Try by yourself

1) Represent on the number line each of the following rational numbers

① $\frac{1}{3}$ ② $\frac{-7}{4}$ ③ $\frac{-1}{3}$ ④ $1\frac{1}{5}$ ⑤ $-3\frac{1}{2}$

2) Compare between

① $\frac{2}{3}$ and $\frac{5}{7}$ 2) $\frac{1}{5}$ and $\frac{1}{6}$ 3) $-\frac{8}{15}$ and $-\frac{2}{3}$

4) 0.6 and $\frac{5}{6}$

3) find two rational numbers lying between

① $\frac{4}{5}$ and $\frac{3}{4}$ 2) $\frac{4}{5}$ and $\frac{5}{6}$ ③ $\frac{4}{5}$ and 0.7 ④ $\frac{3}{4}$, 2

4) Represent the rational numbers: 2, $\frac{-5}{2}$, $\frac{7}{2}$, 0, -1 on the number line then arrange the ascendingly

(5) Complete each of the following using the suitable Sign (<, > or =):

(1) $\frac{3}{4}$ ... $\frac{1}{5}$ (2) $\frac{-3}{15}$ ... $\frac{-2}{5}$ (3) $\frac{3}{15}$ ... $\frac{4}{20}$

(4) $\frac{-6}{9}$ ... $\frac{-2}{6}$ (5) $\frac{3}{2}$ ... $\frac{3}{5}$

(6) Complete each of the following

(1) $\frac{3}{5} > ... > \frac{2}{5}$ (2) $-\frac{1}{3} > ... > -\frac{2}{3}$

(3) $\frac{1}{4} > ... > \frac{1}{8}$ (4) $\frac{-3}{14} > ... > \frac{-2}{7}$

(5) $\frac{1}{4} > ... > \frac{1}{6}$ (6) $-\frac{2}{9} > ... > -\frac{5}{6}$

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Adding and subtracting Rational Numbers

[1] Adding two rational numbers having the same denominator:

If $\frac{a}{b}$ and $\frac{c}{b}$ are two rational numbers,

then: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

[2] Adding two rational numbers with different denominators:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers

then: $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.

Example ① Add:

[1] $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}$

[2] $\frac{5}{6} + (-\frac{1}{6}) = \frac{5-1}{6} = \frac{4}{6} = \frac{2}{3}$ $(\div 2)$

[3] $\frac{7}{8} + (\frac{-5}{8}) = \frac{7-5}{8} = \frac{2}{8} = \frac{1}{4}$ $(\div 2)$

[4] $-\frac{1}{4} + \frac{3}{4} = \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2}$ $(\div 2)$

[5] $\frac{5}{9} + (-\frac{2}{9}) = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$ $(\div 3)$

[6] $-\frac{2}{3} + (-\frac{1}{3}) = \frac{-2-1}{3} = \frac{-3}{3} = -1$

*N

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(7) $\frac{7}{8} - \left(-\frac{3}{8}\right) = \frac{7+3}{8} = \frac{10}{8} = \frac{5}{4}$

(8) $\frac{3}{5} + \left(-\frac{1}{5}\right) = \frac{3-1}{5} = \frac{2}{5}$

(9) $\frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$

(10) $\frac{1}{2} + \frac{1}{3} = \frac{1\times3+1\times2}{2\times3} = \frac{3+2}{6} = \frac{5}{6}$

(11) $\frac{2}{3} + \frac{1}{4} = \frac{2\times4+3\times1}{3\times4} = \frac{8+3}{12} = \frac{11}{12}$

(12) $\frac{1}{2} + \frac{2}{5} = \frac{1\times5+2\times2}{2\times5} = \frac{5+4}{10} = \frac{9}{10}$

(13) $\frac{3}{4} - \frac{1}{3} = \frac{3\times3-1\times4}{4\times3} = \frac{9-4}{12} = \frac{5}{12}$

(14) $-\frac{2}{3} - \frac{1}{2} = \frac{-2\times2-1\times3}{3\times2} = \frac{-4-3}{6} = \frac{-7}{6}$

(15) $-\frac{1}{5} + \frac{1}{3} = \frac{-1\times3+5\times1}{5\times3} = \frac{-3+5}{15} = \frac{2}{15}$

(16) $\frac{3}{4} - \frac{1}{2} = \frac{3\times2-1\times4}{4\times2} = \frac{6-4}{8} = \frac{2}{8} = \frac{1}{4}$

(17) $\frac{4}{9} - \frac{2}{3} = \frac{4\times3-2\times9}{9\times3} = \frac{12-18}{27} = \frac{-6}{27} = \frac{-2}{9}$

Properties of addition operation in $\mathbb{Q}$:

[1] Closure property:

The sum of any two rational numbers is a rational number

If $\frac{a}{b}, \frac{c}{d} \in \mathbb{Q}$ then

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd} \in \mathbb{Q}$$

[2] Commutative property:

If: $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$$

[3] Associative property:

If: $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then:

$$\left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} + \frac{c}{d} + \frac{e}{f}$$

[4] The existence of identity element property in addition:

Zero is the identity element in addition operation in $\mathbb{Q}$

$$\frac{a}{b} + \text{Zero} = \text{Zero} + \frac{a}{b} = \frac{a}{b}$$

(5) the existence of additive inverse property

For every rational number $\frac{a}{b}$ there exist an additive inverse to that is $-\frac{a}{b}$

where $\frac{a}{b} + (-\frac{a}{b}) = \text{Zero}$

$-\frac{a}{b} + \frac{a}{b} = \text{Zero}$

* Zero is its own additive inverse.

Ex:- Complete

① The additive inverse of $\frac{2}{3}$ is ---

② The additive inverse of $-\frac{1}{2}$ is ...

③ the additive inverse of Zero is ---

④ the additive inverse of -1 is ---

⑤ the additive inverse of $(-\frac{1}{2})^{\text{Zero}}$ = ---

⑥ the additive inverse of $(\frac{3}{5})^{\text{Zero}}$ = ---

⑦ the additive inverse of $|-\frac{3}{5}|$ is ---

Ex: Use the addition properties in $\mathbb{Q}$ to Carry out the following:

① $\frac{2}{7} + \frac{3}{4} + \frac{5}{7} + \frac{1}{4}$

$= \left(\frac{2}{7} + \frac{5}{7}\right) + \left(\frac{3}{4} + \frac{1}{4}\right)$

Commutative and associative properties

$= \frac{7}{7} + \frac{4}{4}$

$= 1 + 1 = 2$ (the result in its simplest form)

② $\frac{2}{3} + \frac{5}{9} + \frac{-2}{3} + \frac{4}{9}$

$= \left[\frac{2}{3} + \left(-\frac{2}{3}\right)\right] + \frac{5}{9} + \frac{4}{9}$

Commutative and associative properties

$= 0 + \frac{9}{9}$ (the additive inverse)

$= \frac{9}{9}$ (the identity element)

$= 1$ (the result in its simplest form)

Ex: Write the addition property which used in each of the following

① $\frac{7}{2}+\frac{9}{16}=\frac{9}{16}+\frac{7}{2}$ ② $zero+\frac{3}{4}=\frac{3}{4}$

③ $(\frac{2}{3}+\frac{1}{3})+\frac{1}{6}=\frac{2}{3}+(\frac{1}{3}+\frac{1}{6})$

④ $\frac{3}{4}+(-\frac{3}{4})=zero$ ⑤ $\frac{5}{8}+zero=\frac{5}{8}$

Second: Subtraction operation:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then:

$$\frac{a}{b}-\frac{c}{d}=\frac{a}{b}+(-\frac{c}{d})$$

i.e. the subtraction operation in Q is defined as adding the minued $(\frac{a}{b})$ to the additive inverse of the subtrahend $(\frac{c}{d})$

Ex: If $a = \frac{3}{4}$, $b = \frac{-5}{2}$ and $c = \frac{1}{2}$ find the numerical value of each of:

① $a - b$ ② $(a+b) - c$

Solution

1) $a - b = \frac{3}{4} - \left(\frac{-5}{2}\right) = \frac{3}{4} + \frac{5}{2}$

$= \frac{6+20}{8} = \frac{26}{8} = \frac{13}{4}$

2) $(a+b) - c$

$= \left(\frac{3}{4} + \frac{-5}{2}\right) - \frac{1}{2}$

$= \left(\frac{6-20}{8}\right) - \frac{1}{2}$

$= \frac{-14}{8} - \frac{1}{2} = -\frac{7}{4} - \frac{1}{2} = \frac{-14-4}{8} = \frac{18}{8} = \frac{9}{4}$

Multiplying and Dividing Rational Numbers

First: Multiplication operation

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers

then: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Remember:

$(+) \times (+) = (+)$, $(-) \times (-) = (+)$

$(+) \times (-) = (-)$, $(-) \times (+) = (-)$

Ex: Find the result of each of the following in its simplest form:

① $\frac{3}{6} \times \frac{2}{5} = \frac{3 \times 2}{6 \times 5} = \frac{6}{30} = \frac{1}{5}$

② $\frac{-3}{4} \times \frac{2}{9} = \frac{-3 \times 2}{4 \times 9} = \frac{-6}{36} = \frac{-1}{6}$

③ $\frac{1}{2} \times (-2) = \frac{1 \times -2}{2 \times 1} = \frac{-2}{2} = -1$

④ $\frac{-7}{8} \times \frac{4}{14} = \frac{-7 \times 4}{8 \times 14} = \frac{-1 \times 1}{2 \times 2} = \frac{-1}{4}$

⑤ $25\% \times (\frac{-3}{4}) = \frac{25}{100} \times (\frac{-3}{4}) = \frac{1}{4} \times \frac{-3}{4} = \frac{-3}{16}$

⑥ $-1\frac{1}{2} \times -0.\dot{3} = -\frac{3}{2} \times -\frac{1}{3} = \frac{-3 \times -1}{2 \times 3} = \frac{3}{6} = \frac{1}{2}$

properties of the Set of rational numbers under multiplication:

for any three rational numbers as $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$

the following properties are satisfied :

(1) Closure property:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \in Q$$

The product of any two rational numbers is arational number.

(2) Commutative property:

$$\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

(3) Associative property:

$$\left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} \times \frac{e}{f}$$

(4) The existence of multiplicative indentity element property:

The number 1 "one" is the multiplicative identity in Q

$$\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}$$

(5) The existence of multiplicative inverse property in Q
for every rational number $\frac{a}{b}$ except zero

there is a multiplicative invers $\Rightarrow \frac{b}{a}$

$$\frac{b}{a} \times \frac{a}{b} = 1$$

the multiplicative inverse property

Ex: Complete

① The multiplicative inverse of the number $\frac{3}{4}$ is ...

② The multiplicative inverse of the number -5 is ...

③ the multiplicative inverse of the number 0.5 is ...

④ the multiplicative inverse of the number $2\frac{1}{2}$ ...

⑤ the multiplicative inverse of the number $(-\frac{2}{3})^{zero}$ ...

⑥ the multiplicative inverse of the number -1 is ...

Remarks

① The multiplicative inverse of the rational number is called the reciprocal of the rational number

② Zero has no multiplicative inverse

$\frac{1}{0}$ is meaningless

③ multiplying any rational number by zero equals zero



6 The distributive property:

Ex: Use the distributing property to find the value of each of the following:

1. $\frac{13}{12} \times 7 + \frac{13}{12} \times 5$

$= \frac{13}{12} \times (7 + 5) = \frac{13}{12} \times \frac{12}{1} = 13$

2. $\frac{4}{9} \times 11 + \frac{4}{9} \times 16$

$= \frac{4}{9} \times (11 + 16) = \frac{4}{9} \times \frac{27}{1} = 12$

3. $\frac{5}{13} \times 2 + \frac{5}{13} \times 4 + \frac{5}{3} \times 7$

$= \frac{5}{13} \times (2 + 4 + 7) = \frac{5}{13} \times 13 = 5$

4. $\frac{4}{7} \times 4 + \frac{4}{7} \times 2 + \frac{4}{7} \times 1$

$= \frac{4}{7} \times (4 + 2 + 1)$

$= \frac{4}{7} \times 7 = 4$

5. $\frac{-3}{7} \times 8 + 5 \times (\frac{-3}{7}) + (\frac{-3}{7})$

$= \frac{-3}{7} \times (8 + 5 + 1) = \frac{-3}{7} \times 14 = -6$

Second: Division operation:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Ex: Find the result of each of the following in the simplest form:

① $\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$

② $\frac{-3}{5} \div \frac{9}{5} = \frac{-3}{5} \times \frac{5}{9} = \frac{-15}{45} = -\frac{1}{3}$

③ $\frac{-14}{15} \div \left(\frac{-21}{5}\right) = \frac{-14}{15} \times \frac{-5}{21} = \frac{-2}{3} \times \frac{-1}{3} = \frac{2}{9}$

④ $2\frac{1}{4} \div 1\frac{1}{2} = \frac{9}{4} \div \frac{3}{2} = \frac{9}{4} \times \frac{2}{3} = \frac{18}{12} = \frac{3}{2}$

Ex: find the value of x in each of the following

① $\frac{5}{7} \times x = \frac{5}{7}$ ② $x \times \frac{17}{3} = 1$

③ $-\frac{7}{3} \times x = \text{zero}$ ④ $-\frac{7}{3} \times -\frac{3}{7} = x$

⑤ $\frac{3}{5} \times x = -\frac{4}{5} \times \frac{3}{5}$

⑥ $\frac{3}{4} \div x = \frac{3}{4} \times -\frac{2}{5}$

Applications on the Rational Numbers

[1] The number that lies at the midpoint of the way between any two numbers

= ½ Sum of them

[2] the number that lies at one third of the way between two numbers

= The smallest number + ⅓ the distance

= the greatest number − ⅓ the distance between the two numbers

Ex: Find the number that lies at the midpoint of the way between $\frac{1}{2}$, $\frac{1}{3}$

Solution

$\frac{1}{2}$, $\frac{1}{3}$ ⇒ $\frac{3}{6}$, $\frac{2}{6}$

the Sum = $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

The number that lies at the midpoint of the way between $\frac{1}{2}$, $\frac{1}{3}$ = ½ sum

$= \frac{1}{2} \times \frac{5}{6}$

$= \frac{5}{12}$

Ex: Find a rational number lying at one fourth of the way between $\frac{1}{2}$, $\frac{1}{3}$

Solution

$$\frac{1}{2}, \frac{1}{3} \Rightarrow \frac{3}{6}, \frac{2}{6}$$

The distance $= \left| \frac{3}{6} - \frac{2}{6} \right| = \frac{1}{6}$

the required number is

the smaller number + $\frac{1}{4}$ distance

$$= \frac{2}{6} + \frac{1}{4} \times \frac{1}{6}$$

$$= \frac{2}{6} + \frac{1}{24} = \frac{8+1}{24} = \frac{9}{24} = \frac{3}{8}$$

Ex: Find a rational number lying at one fifth of the way between $\frac{2}{5}$ and $\frac{4}{7}$

Solution $\frac{2}{5}, \frac{4}{7} \Rightarrow \frac{14}{35}, \frac{20}{35}$

the distance $= \left| \frac{20}{35} - \frac{14}{35} \right| = \frac{6}{35}$

the required number =

the smaller number + $\frac{1}{5}$ distance

$$= \frac{14}{35} + \frac{1}{5} \times \frac{6}{35} = \frac{14}{35} + \frac{6}{175} = \frac{70+6}{175} = \frac{76}{175}$$